Student Number:______________________ Class Teacher: _________________

St George Girls High School

Trial Higher School Certificate Examination

2019

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours.
- Write using black pen.
- Board-approved calculators may be used.
- A reference sheet is provided.
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for incomplete or poorly presented solutions.

Section I	/10
Section II	
Question 11	/10
Question 12	/10
Question 13	/10
Question 14	/10
Question 15	/10
Question 16	/10
Total	**/70**

Total Marks – 70

Section 1 Pages 3 – 6

10 marks

- Attempt Questions 1 – 10.
- Allow about 15 minutes for this section.
- Answer on the multiple choice answer sheet provided at the back of this paper.

Section II Pages 7 – 13

60 marks

- Attempt Questions 11 – 16.
- Allow about 1 hour and 45 minutes for this section.
- Begin each question in a new writing booklet.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 - 10

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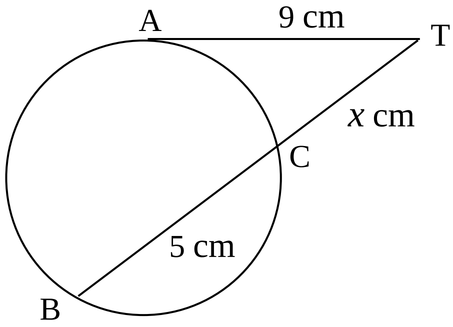
1. The line AT is the tangent to the circle at A and the line BT is a secant meeting the circle at B and C, as shown in the diagram.



Given that $\text{AT} = 9, \text{BC} = 5$ and $\text{CT} = x$, which one of the following equations is correct?

(A) $x^2 + 5x - 81 = 0$

(B) $x^2 + 5x + 81 = 0$

(C) $x^2 - 5x - 81 = 0$

(D) $x^2 + 5x - 9 = 0$

2. The acute angle between the lines $y = 2x + 4$ and $5x - y + 34 = 0$, to the nearest degree is:

(A) $4°$

(B) $7°$

(C) $15°$

(D) $74°$

3. If $t = \tan\frac{\theta}{2}$, what is the correct expression for $\frac{1-\cos\theta}{\sin\theta}$.

(A) $\frac{1}{t}$

(B) t

(C) $2t$

(D) $\frac{2}{t}$

4. Find the Cartesian equation of the curve defined by the parametric equations:

$$x = \sin\theta$$
$$y = \cos^2\theta - 3.$$

(A) $y = -3 + 3x^2$

(B) $y = \sin^2 x - 3$

(C) $y = -2 - x^2$

(D) $y = \sin 2x + 3\cos^2 x$

5. What is the value of $\lim_{x\to 0} \frac{\sin 2x}{x \cos x}$?

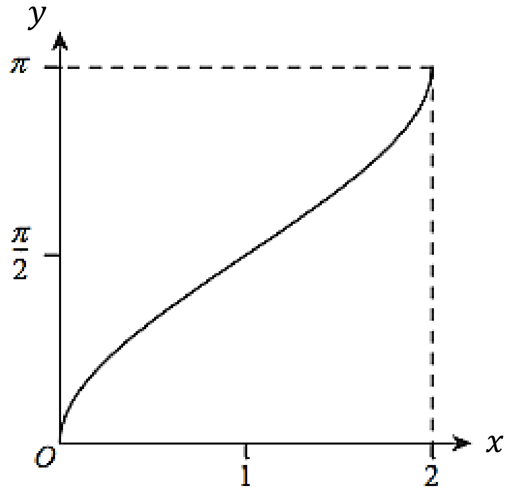
(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

6. Consider the graph below.



Which function best describes this graph?

(A) $y = \cos^{-1}(x)$

(B) $y = 1 - \cos^{-1}(x)$

(C) $y = \cos^{-1}(x - 1)$

(D) $y = \cos^{-1}(1 - x)$

7. $\int 4\cos^2 4x \; dx =$

(A) $\left(2x + \frac{1}{4}\sin 8x\right) + C$

(B) $\left(x + \frac{1}{2}\sin 8x\right) + C$

(C) $\left(x + \frac{1}{2}\cos 8x\right) + C$

(D) $\left(x + \frac{1}{4}\sin 8x\right) + C$

8. The velocity of a particle is given by $v = \sqrt{2-x}$, where x is its displacement in metres and velocity (m/s).
Which of the following is a correct expression for the acceleration $\ddot{x}$?

(A) $\ddot{x} = \frac{1}{2}\,\text{m/s}^2$

(B) $\ddot{x} = \frac{1}{4}\,\text{m/s}^2$

(C) $\ddot{x} = -\frac{1}{2}\,\text{m/s}^2$

(D) $\ddot{x} = -\frac{1}{4}\,\text{m/s}^2$

9. Which of the following is a general solution of the equation $\sin\frac{x}{2} = \sin\frac{\pi}{10}$?

(A) $x = n\pi + (-1)^n\frac{\pi}{5}$

(B) $x = \frac{n\,\pi}{2} + (-1)^n\frac{\pi}{20}$

(C) $x = 2n\pi + (-1)^n\frac{\pi}{10}$

(D) $x = 2n\pi + (-1)^n\frac{\pi}{5}$

10. The cubic curve $y = x^3 + 3ax + b$ has two turning points and crosses the $y-$ axis at $(0, -a)$.
Which of the following could be true?

(A) $a < 0$ and $b > 0$

(B) $a > 0$ and $b < 0$

(C) $a > 0$ and $b > 0$

(D) $a < 0$ and $b < 0$

End of Section I

Section II

60 marks
Attempt Questions 11 – 16
Allow about 1 hour and 45 minutes for this section
Answer each question in the appropriate writing booklet.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 **(10 marks)** **Use a separate writing booklet** **Marks**

(a) The point $C\ (x, y)$ divides the interval joining $A\ (-\ 4, 8)$ to $B\ (6, -12)$ internally in the ratio 2 : 3. **2**

Find the coordinates of C.

(b) Solve for x: $\dfrac{x}{1-3x} \geq 1$. **3**

(c) Show that $\dfrac{\sin 3x}{\sin x} - \dfrac{\cos 3x}{\cos x} = 2$. **2**

(d) Find the exact value of $\displaystyle\int_0^{\frac{\sqrt{5}}{2}} \frac{dx}{\sqrt{5-4x^2}}$. **3**

Question 12 (10 marks) Use a separate writing booklet **Marks**

(a) If α, β and γ are the roots of the equation $2x^3 + 5x - 3 = 0$, find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$. 2

(b) The rate of increase of a population P of sandflies on the track to Culbarra Beach is proportional to the difference between the population, P, and 2000. This rate is expressed by the differential equation $\frac{dP}{dt} = k(P - 2000)$, where k is a constant and t represents time in weeks.

(i) Show that $P = 2000 + Ae^{kt}$, where A is a constant, satisfies the differential equation 1

(ii) Initially, the population was 2500 and two weeks later it had increased to 5000. Find the value of A and k. 2

(iii) After how many weeks will the population of sandflies exceed 10 000? 2

(c) Evaluate $\displaystyle\int_{-1}^{2} x\sqrt{3 - x}\ dx$ using the substitution $u = 3 - x$, $x < 3$. 3

Question 13 **(10 marks)** **Use a separate writing booklet** **Marks**

(a) Consider the continuous function $f(x) = \sec^{-1}x + \sin^{-1}\frac{1}{x}$ for $x \geq 1$.

(i) Show that $\sec^{-1}x = \cos^{-1}\frac{1}{x}$. 1

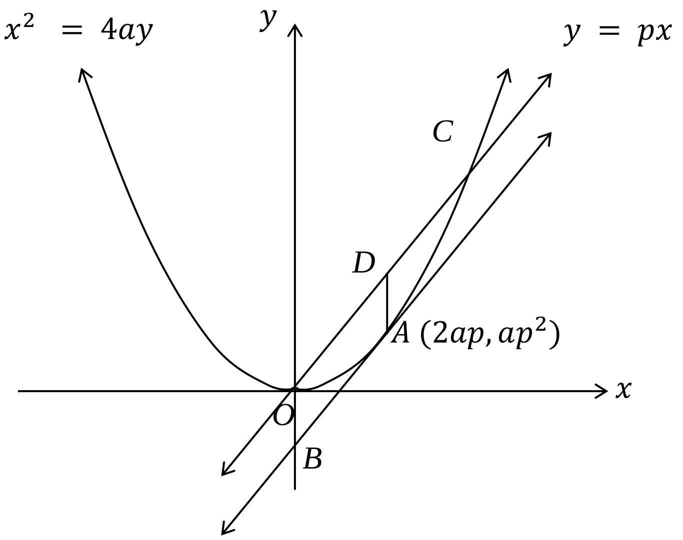
(ii) Hence, show that $f'(x) = 0$. 2

(iii) Hence, or otherwise, prove that 2

$$\sec^{-1}x + \sin^{-1}\frac{1}{x} = \frac{\pi}{2} \text{ for } x \geq 1.$$

(b) The point $A(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$.
The tangent at A intersects the y −axis at B.

The line $y = px$ crosses the parabola at the origin and at the point C.
Let D be the midpoint of OC.



(i) Show that the coordinates of D is $(2ap, 2ap^2)$. 1

(ii) Show that the coordinates of B is $(0, -ap^2)$. 2

(iii) Show that $ODAB$ is a parallelogram. 2

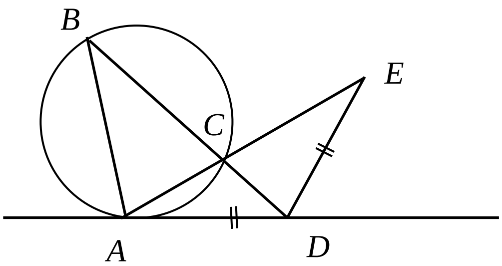
Question 14 **(10 marks)** **Use a separate writing booklet** **Marks**

(a) Consider the function $f(x) = 3\sin 2x - x$.

(i) Show that $f(x) = 0$ has a root α such that $1.33 < \alpha < 1.34$. 2

(ii) Starting with $\alpha = 1.33$, use one application of Newton's Method to find a better approximation for this root, giving your answer to 4 decimal places. 2

(b)



In the diagram, ABC is a triangle inscribed in the circle. The tangent to the circle at A meets BC produced to D. E is the point on AC produced such that $DA = DE$. 3

Copy the diagram into your booklet.

Prove that $ABED$ is a cyclic quadrilateral.

(c) Use mathematical induction to prove that $4^n + 6n - 1$ is divisible by 9 for integers $n \geq 1$. 3

Question 15 (10 marks) Use a separate writing booklet **Marks**

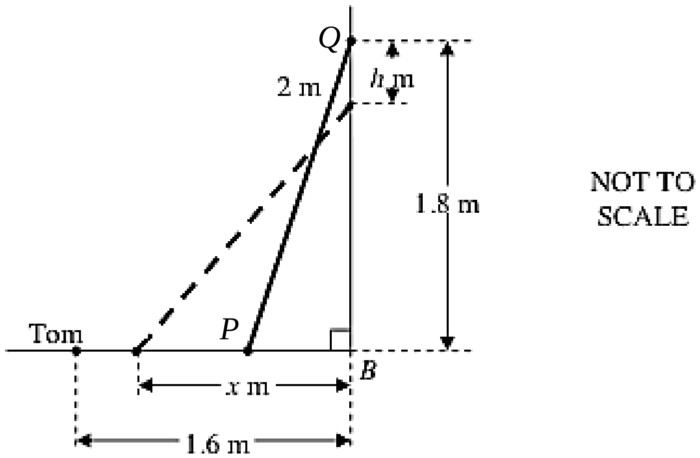
(a) A particle is moving so that its distance x centimetres from a fixed point O at time t seconds is $x = 6\sin 2t$.

(i) Show that the particle is moving in simple harmonic motion. 2

(ii) Find the period of the motion. 1

(iii) Find the velocity of the particle when it first reaches 3 centimetres to the right of the origin. 2

(b) The diagram shows a ladder PQ, 2 metres in length, leaning against a wall such that the top of the ladder, Q, initially reaches 1.8 metres up the wall. The base of the ladder, P, is x metres from the base of the wall, B.



The ladder begins to slide down the wall at the rate of 0.5 metres per minute such that the top of the ladder is h metres below its original position after t minutes.

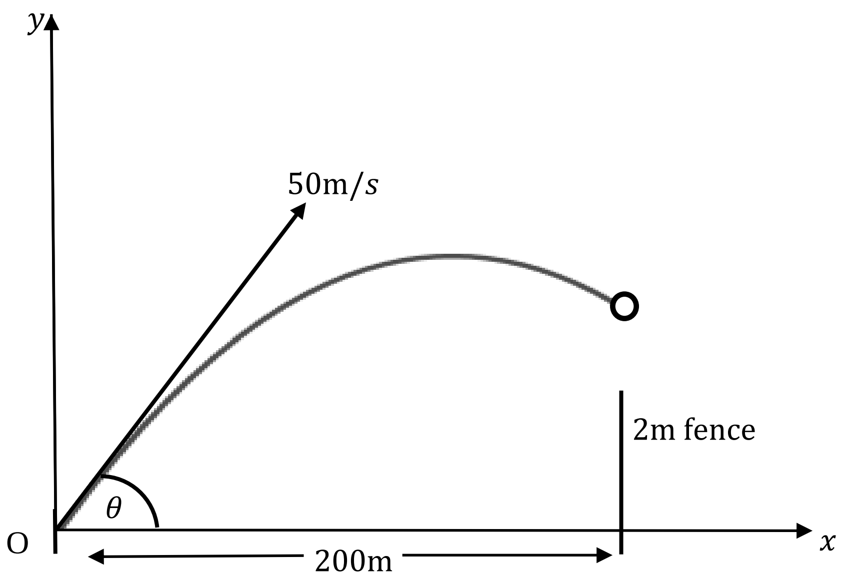
(i) Show that t minutes after the ladder begins to slide down the wall, 2

$$h = 1.8 - \sqrt{4 - x^2}.$$

(ii) Tom is standing on the ground 1.6 metres from the base of the wall in a direct line with the ladder. At what rate does base of the ladder hit Tom? 3

Question 16 **(10 marks)** **Use a separate writing booklet** **Marks**

(a) A method to score a home run in a baseball game is to hit the ball over the boundary fence on the full.



A ball is hit at 50 m/s The fence, 200 metres away, is 2 metres high. You may neglect air resistance and acceleration due to gravity can be taken as 10 m/s^2. You may assume the following equations of motion:

$$x = 50t\cos\theta \quad \text{and} \quad y = 50t\sin\theta - 5t^2$$

DO NOT PROVE THESE EQUATIONS

(i) Show that the Cartesian equation of motion is given by **2**

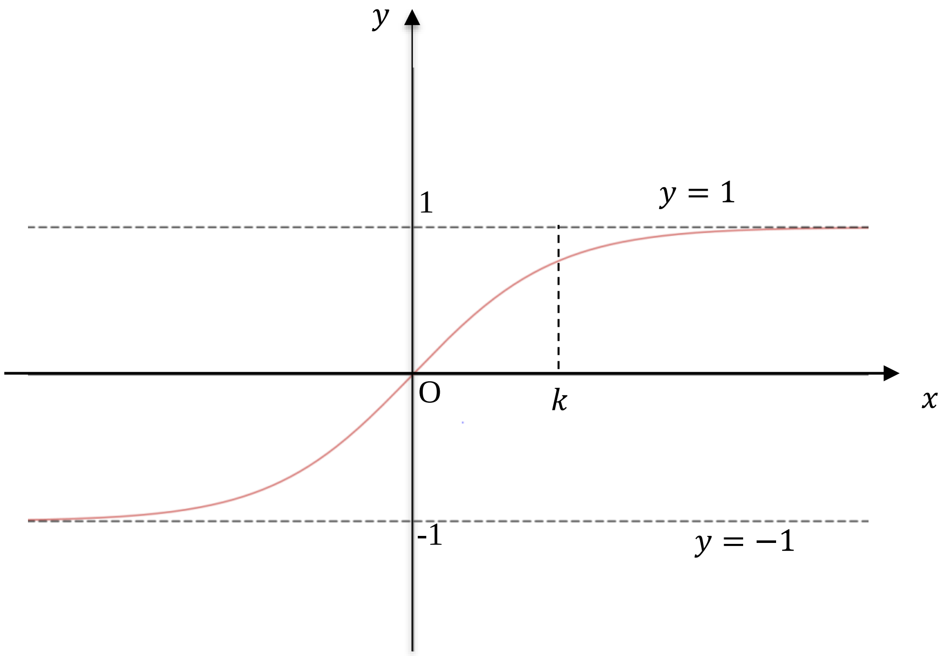
$$y = x\tan\theta - \frac{x^2}{500}(\sec^2\theta), \quad \text{where } \theta \text{ is the angle of projection.}$$

(ii) Show that if the ball just clears the 2 metre boundary fence then, **2**

$$40\tan^2\theta - 100\tan\theta + 41 = 0.$$

(iii) In what range of values must θ lie to score a home run by this method? **2**

(b)



The curve shows the graph of the function

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

where $y = \pm 1$ are the horizontal asymptotes.

(i) If k is a positive constant, show that the area in the first quadrant enclosed by the above curve, the lines $y = 1, x = 0$ and $x = k$ is given by: **2**

$$A = k - \ln(e^k + e^{-k}) + \ln 2$$

(ii) By considering the area in (i), prove that for all positive values of k, the area is always less than $\ln 2$. **2**

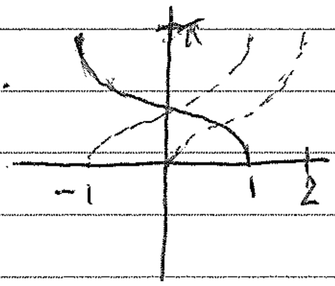
END OF PAPER

Year 12 Ext 1 Trial HSC - 2019 Solutions

MATHEMATICS EXTENSION 1 – QUESTION Multiple Choice

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
1. $9^2 = x(x+5)$		
$81 = x^2 + 5x$		
$x^2 + 5x - 81 = 0$	A	
2. For $y = 2x + 4$, $m_1 = 2$		
$5x - y + 34 = 0$, $m_2 = 5$		
$\tan\theta = \left\lvert \dfrac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$		
$= \left\lvert \dfrac{2 - 5}{1 + 2(5)} \right\rvert$		
$= \dfrac{3}{11}$		
$\theta \doteqdot 15^\circ$	C	
3. $\dfrac{1 - \cos\theta}{\sin\theta} = \dfrac{1 - \dfrac{1-t^2}{1+t^2}}{\dfrac{2t}{1+t^2}}$ $\begin{matrix} \times 1+t^2 \\ \times 1+t^2 \end{matrix}$		
$= \dfrac{1 + t^2 - (1 - t^2)}{2t}$		
$= \dfrac{1 + t^2 - 1 + t^2}{2t}$		
$= \dfrac{2t^2}{2t}$		
$= 1$	B	

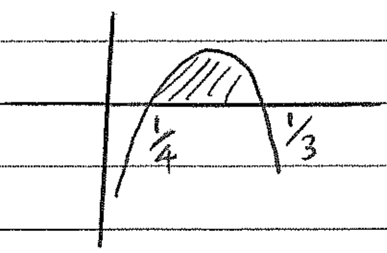
MATHEMATICS EXTENSION 1 – QUESTION MC

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
4. $x = \sin\theta$ — — (1)		
$y = \cos^2\theta - 3$ — — (2)		
From (2) $y = (1 - \sin^2\theta) - 3$		
$= -\sin^2\theta - 2$		
$= -x^2 - 2$ from (1)	C	
5. $\lim_{x\to 0} \frac{\sin 2x}{x\cos x} = \lim_{x\to 0} \frac{2\sin x\cos x}{x\cos x}$		
$= 2\lim_{x\to 0} \frac{\sin x}{x}$		
$= 2 \times 1$		
$= 2$	D	
6.  $y = \cos^{-1} x$		
$y = \cos^{-1}(-x)$		
$y = \cos^{-1}(-x+1)$	D	
$y = \cos^{-1}(1-x)$		
7. $\int 4\cos^2 4x \, dx$		
$= 4\int \cos^2 4x \, dx$		
$= 4\int \frac{1}{2}(\cos 8x + 1) \, dx$		
$= 2\left[\frac{\sin 8x}{8} + x\right) + c$		
$= 2x + \frac{1}{4}\sin 8x + c$	A	

MATHEMATICS EXTENSION 1 – QUESTION MC

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
8. $v = \sqrt{2-x}$		
$v^2 = 2-x$		
$\ddot{x} = \dfrac{d\left(\frac{1}{2}v^2\right)}{dx}$		
$= \dfrac{d\left(\frac{1}{2}(2-x)\right)}{dx}$		
$= \dfrac{d\left(1-\frac{x}{2}\right)}{dx}$		
$= -\frac{1}{2}\ m/s^2$	C	
9. $\sin\frac{x}{2} = \sin\frac{\pi}{10}$		
$\frac{x}{2} = \sin^{-1}\left(\sin\frac{\pi}{10}\right)$		
$\frac{x}{2} = \frac{\pi}{10}$		
$\frac{x}{2} = n\pi + (-1)^n\alpha$		
$\frac{x}{2} = n\pi + (-1)^n\frac{\pi}{10}$		
$x = 2n\pi + (-1)^n\frac{\pi}{5}$	D	
10. $y' = 3x^2 + 3a$		
For tp, $y' = 0$		
$3x^2 + 3a = 0$		
$x^2 + a = 0$		
Two real solutions if $a<0$. As curve passes		
through $(0,-a)$ then $y=-a$ and $x=0$.		
So $-a = 0+b$		
$b = -a$ So $a<0$ then $b>0$	A	

MATHEMATICS EXTENSION I – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $\overset{x_1,\ y_1}{A(-4, 8)}$ $\overset{x_2\ \ y_2}{B(6, -12)}$		
$2 : 3$		
$m : n$		
$x = \dfrac{nx_1 + mx_2}{m+n}$ $\quad y = \dfrac{ny_1 + my_2}{m+n}$		2 marks for both values of x and y i.e $C(0,0)$
$= \dfrac{3(-4) + 2(6)}{2+3}$ $\quad = \dfrac{3(8) + 2(-12)}{2+3}$		
$= \dfrac{-12 + 12}{5}$ $\quad = \dfrac{24 - 24}{5}$		
$= 0$ $\quad = 0$		1 mark for either x or y correctly found.
$\therefore$ c is $(0,0)$		
b) $\dfrac{x}{1-3x} \geq 1$, $x \neq \dfrac{1}{3}$		
Multiplying both sides by $(1-3x)^2$		
$(1-3x)^2 \cdot \dfrac{x}{(1-3x)} \geq 1 \times (1-3x)^2$		
$(1-3x)x \geq (1-3x)^2$		
$x(1-3x) - (1-3x) \geq 0$	1	
$(1-3x)[x - (1-3x)] \geq 0$		
$(1-3x)(4x-1) \geq 0$ $\quad$ --- (*)	1	
		Many students had problems drawing the quadratic. Either the concavity was incorrect or the x-intercepts were not labelled correctly
$\therefore$ $\dfrac{1}{4} \leq x < \dfrac{1}{3}$	1	

MATHEMATICS EXTENSION I – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Furthermore for Q11(b)		
• some students included $\frac{1}{3}$ in their solution and gave $\frac{1}{4} \le x \le \frac{1}{3} \Rightarrow$		$2\frac{1}{2}$ marks were awarded
• Many students used the incorrect sign to the quadratic inequality and therefore the solution obtained was $x \le \frac{1}{4}$ or $x > \frac{1}{3}$		2 marks were awarded
11(c)		
$\text{LHS} = \frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$		
$= \frac{\cos x \sin 3x - \sin x \cos 3x}{\sin x \cos x}$	½	
$= \frac{\sin 3x \cos x - \cos 3x \sin x}{\sin x \cos x}$		1 mark
$= \frac{\sin(3x - x)}{\sin x \cos x}$		
$= \frac{\sin 2x}{\sin x \cos x}$		1 mark.
$= \frac{2 \sin x \cos x}{\sin x \cos x}$		
$= 2$		
$= \text{RHS}$		

MATHEMATICS EXTENSION I – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
NOTE:		
Many students thought ~~that~~ they could factorise,		
$\cos x \sin 3x - \sin x \cos 3x$ as $\cos x \sin x(\sin 2x - \cos x)$		
This is quite concerning and care needs to be taken.		
Please revise compound angles.		
Alternative solution (by many students) (but not preferred)		
$LHS = \dfrac{\sin 3x}{\sin x} - \dfrac{\cos 3x}{\cos x}$		
$= \dfrac{\sin(2x+x)}{\sin x} - \dfrac{\cos(2x+x)}{\cos x}$		
$= \dfrac{\sin 2x \cos x + \cos 2x \sin x}{\sin x} - \dfrac{\cos 2x \cos x - \sin 2x \sin x}{\cos x}$	$\frac{1}{2}$	
$= \dfrac{\sin 2x \cos x}{\sin x} + \dfrac{\cos 2x \cancel{\sin x}}{\cancel{\sin x}} - \dfrac{\cos 2x \cancel{\cos x}}{\cancel{\cos x}} + \dfrac{\sin 2x \sin x}{\cos x}$	$\frac{1}{2}$	
$= \dfrac{2\cancel{\sin x}\cos x \cos x}{\cancel{\sin x}} + \dfrac{2\sin x \cancel{\cos x} \sin x}{\cancel{\cos x}}$	$\frac{1}{2}$	
$= 2\cos^2 x + 2\sin^2 x$	$\frac{1}{2}$	
$= 2(\cos^2 x + \sin^2 x)$		
$= 2$		
$= RHS$		

MATHEMATICS EXTENSION I – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) $\displaystyle\int_0^{\frac{\sqrt{5}}{2}} \frac{dx}{\sqrt{5-4x^2}}$		
$= \displaystyle\int_0^{\sqrt{5}/2} \frac{dx}{\sqrt{4\left(\frac{5}{4}-x^2\right)}}$	$\frac{1}{2}$	Many students took 2 out of the integrand instead of $\frac{1}{2}$. $\therefore$ $2\frac{1}{2}$ marks awarded if answer was π. Similarly if nothing was taken out, $2\frac{1}{2}$ marks were awarded if the remaining solution was correct.
$= \displaystyle\frac{1}{2}\int_0^{\sqrt{5}/2} \frac{dx}{\sqrt{\frac{5}{4}-x^2}}$ Many students took 2 out of the integrand instead of $\frac{1}{2}$		
$= \displaystyle\frac{1}{2}\int_0^{\frac{\sqrt{5}}{2}} \frac{dx}{\sqrt{\left(\frac{\sqrt{5}}{2}\right)^2-x^2}}$	$\frac{1}{2}$	
$= \displaystyle\frac{1}{2}\left[\sin^{-1}\frac{x}{\frac{\sqrt{5}}{2}}\right]_0^{\sqrt{5}/2}$	1	
$= \displaystyle\frac{1}{2}\left[\sin^{-1}\frac{2x}{\sqrt{5}}\right]_0^{\sqrt{5}/2}$		
$= \displaystyle\frac{1}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{2} - \sin^{-1}0\right]$	$\frac{1}{2}$	
$= \displaystyle\frac{1}{2}\sin^{-1}1$		
$= \displaystyle\frac{1}{2}\times\frac{\pi}{2}$		
$= \pi/4$	$\frac{1}{2}$	
Note: Some students did not find the definite integral and just integrated to give $\frac{1}{2}\sin^{-1}\frac{2x}{\sqrt{5}}+c$ $\Rightarrow$	1 mark awarded.	

MATHEMATICS EXTENSION 1 – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $2x^3 + 5x - 3 = 0$ $a = 2$, $b = 0$, $c = 5$, $d = -3$		*Many students missed this connection, and overcomplicated the question.
$\boxed{\alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}}$ *		
$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \alpha\gamma + \beta\gamma}{\alpha\beta\gamma}$		
$= \frac{c/a}{-d/a}$		
$= \frac{5/2}{3/2}$	1	for either $5/2$ or $3/2$
$= \frac{5}{3}$	1	
Note: an answer of $\frac{-5}{3}$ was awarded $1\frac{1}{2}$ marks.		
b) i If $P = 2000 + Ae^{kt}$ then $\frac{dP}{dt} = kAe^{kt}$ $= k(2000 + Ae^{kt} - 2000)$ $= k(P - 2000)$	1	
so it satisfies the differential equation		
Note: In a "show" question, you must show every step. Do not expect the examiner to join the dots for you. Many students left out crucial steps or substitutions, and only earned half marks.		

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) i – Alternative solution		* crucial step
If $P = 2000 + Ae^{kt}$, then $Ae^{kt} = P - 2000$* ①		
$\frac{dP}{dt} = kAe^{kt}$		1 mark
$= k(P-2000)$ (from ①)		
so it satisfies the differential equation		
ii when $t=0$, $P=2500$		
$\therefore 2500 = 2000 + Ae^{0k}$		
$= 2000 + A$		
$\therefore A = 500$	1	
when $t=2$, $P=5000$		
$\therefore 5000 = 2000 + 500e^{2k}$		
$500e^{2k} = 3000$		
$e^{2k} = 6$		
$2k = \ln 6$		
$k = \frac{1}{2}\ln 6$	1	
$= 0.895879\ldots$		
$\doteqdot 0.896$ (3 d.p.)		
iii Population will reach 10000 when		
$10000 = 2000 + 500e^{kt}$	1	
$500e^{kt} = 8000$		
$e^{kt} = 16$		
$kt = \ln 16$		

MATHEMATICS EXTENSION 1 – QUESTION 12 (continued)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$t = \frac{\ln 16}{k}$ but $k = \frac{1}{2}\ln 6$		
$= \frac{\ln 16}{\frac{1}{2}\ln 6}$		
$= \frac{2\ln 16}{\ln 6}$		
$= 3.0948224\ldots$	1	
$\therefore$ the poplation will exceed 10000 during the 4th week.		
c) $\int_{-1}^{2} x\sqrt{3-x}\,dx$ $\quad u = 3-x \Rightarrow x = 3-u$, $\frac{du}{dx} = -1$, $dx = -du$		
when $x=-1$, $u=4$; when $x=2$, $u=1$	1	
$= \int_{4}^{1} (3-u)\sqrt{u}\,(-du)$	1	Candidates who mistakenly made the integration easier could not earn the 3rd mark.
$= \int_{1}^{4} 3\sqrt{u} - u\sqrt{u}\,du$		
$= \left[2u^{\frac{3}{2}} - \frac{2u^{\frac{5}{2}}}{5}\right]_1^4$		
$= \frac{16}{5} - \frac{8}{5}$		

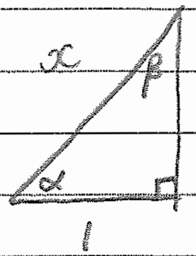
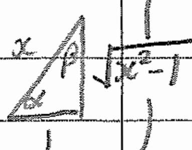
$= \frac{8}{5}$ $\qquad$ 1

MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a)(i) let $\alpha = \sec^{-1} x$		
$\sec\alpha = x$		Poorly attempted
$\cos\alpha = \frac{1}{x}$	1	NOTE:
$\therefore \alpha = \cos^{-1}\frac{1}{x}$		$\sec^{-1}x \neq (\sec x)^{-1}$
$= RHS$		
OR $LHS = \cos^{-1}\frac{1}{x}$		
let $\alpha = \cos^{-1}\frac{1}{x}$		
$\cos\alpha = \frac{1}{x}$		
$\sec\alpha = x$		
$\therefore \alpha = \sec^{-1}x$		
(ii) $f(x) = \sec^{-1}x + \sin^{-1}\frac{1}{x}$		
$= \cos^{-1}\frac{1}{x} + \sin^{-1}\frac{1}{x}$ from (i)		
Consider $\frac{d}{dx}\left(\sin^{-1}\frac{1}{x}\right)$ let $u = \frac{1}{x}$		
$u = x^{-1}$		
$= \frac{d}{dx}\left(\sin^{-1}u\right)$ $\frac{du}{dx} = -x^{-2}$		
$= -\frac{1}{x^2}$		
$= \frac{1}{\sqrt{1-u^2}}$		
$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-u^2}} \times -\frac{1}{x^2}$		
$= \frac{-1}{x^2\sqrt{1-\left(\frac{1}{x}\right)^2}}$		
$= \frac{-1}{x^2\sqrt{\frac{x^2-1}{x^2}}}$		
$= \frac{-1}{\frac{x^2}{x}\sqrt{x^2-1}}$		
$= \frac{-1}{x\sqrt{x^2-1}}$	1	

MATHEMATICS EXTENSION I – QUESTION 13		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Similarly $\frac{d}{dx}\left(\cos^{-1}\frac{1}{x}\right)$		
$= -\left(-\frac{1}{x\sqrt{x^2-1}}\right)$		
$= \frac{1}{x\sqrt{x^2-1}}$	1	
$\therefore \quad f(x) = \cos^{-1}\left(\frac{1}{x}\right) + \sin^{-1}\left(\frac{1}{x}\right)$		
$f'(x) = \frac{1}{x\sqrt{x^2-1}} + \frac{-1}{x\sqrt{x^2-1}}$		
$f'(x) = 0$		
(iii) Since the gradient is always zero for $x \geq 1$ (ie $f'(x) = 0$ for $x \geq 1$) for $f(x)$, then $f(x)$ must be a horizontal line for $x \geq 1$ (ie $f(x)$ must be a constant in the domain $x \geq 1$)	1	
$\therefore$ You can sub any x-value, $x \geq 1$ to find $f(x)$.		
$f(1) = \sec^{-1}(1) + \sin^{-1}\left(\frac{1}{1}\right)$		
$= \cos^{-1}\left(\frac{1}{1}\right) + \sin^{-1}(1)$		
$= \cos^{-1}(1) + \sin^{-1}(1)$		
$= 0 + \frac{\pi}{2}$	1	
$= \frac{\pi}{2}$		
Hence $f(x) = \sec^{-1}\frac{1}{x} + \sin^{-1}\frac{1}{x}$		
$= \frac{\pi}{2}$		

MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
OR $f(x) = \sec^{-1}x + \sin^{-1}\frac{1}{x}$		
$f(x) = \cos^{-1}\frac{1}{x} + \sin^{-1}\frac{1}{x}$		
let $\alpha = \cos^{-1}\frac{1}{x}$ $\quad$ $\beta = \sin^{-1}\frac{1}{x}$		Using one triangle to represent the information.
$\cos\alpha = \frac{1}{x}$ $\quad$ $\sin\beta = \frac{1}{x}$	1	
		
$\therefore\ \alpha + \beta + \frac{\pi}{2} = \pi$ (angle sum ~~right-angled triangle~~)	1	Using < sum Δ of a triangle to prove identity.
$\alpha + \beta = \pi - \frac{\pi}{2}$		
$\alpha + \beta = \frac{\pi}{2}$		
OR $f(x) = \sec^{-1}x + \sin^{-1}\frac{1}{x}$		
$f(x) = \cos^{-1}\frac{1}{x} + \sin\frac{1}{x}$		
To Prove $\cos^{-1}\frac{1}{x} + \sin^{-1}\frac{1}{x} = \frac{\pi}{2}$		
$\sin\left(\cos^{-1}\frac{1}{x} + \sin^{-1}\frac{1}{x}\right) = \sin\frac{\pi}{2} = 1$	1	Using sin correctly
$LHS = \sin\left[\cos^{-1}\frac{1}{x} + \sin^{-1}\frac{1}{x}\right]$ $\quad$ let $\alpha = \cos^{-1}\frac{1}{x}$, $\cos\alpha = \frac{1}{x}$ 		
$= \sin(\alpha + \beta)$ $\quad$ $\beta = \sin^{-1}\frac{1}{x}$		
$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad$ $\sin\beta = \frac{1}{x}$		
$= \frac{\sqrt{x^2-1}}{x} \times \frac{\sqrt{x^2-1}}{x} + \frac{1}{x} \times \frac{1}{x}$	1	correct application
$= \frac{x^2-1}{x^2} + \frac{1}{x^2}$		
$= \frac{x^2}{x^2}$		
$= 1$		
$= RHS.$ $\quad$ and similarly using cos or tan		

MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) $x^2 = 4ay \qquad y = px$		
Solve simultaneously		
$x^2 = 4a(px)$		
$x^2 = 4apx$		
$x^2 - 4apx = 0$		Make sure
$x(x - 4ap) = 0$		you do not
$x = 0 \quad$ or $\quad x = 4ap$		÷ by x. (not penalised)
sub $x = 4ap$ into $y = px$		
$O(0,0) \qquad = p(4ap)$		Show substitution
$= 4ap^2$	½	(not penalised)
$\therefore C(4ap, 4ap^2)$		
Midpoint $\quad x = \frac{0 + 4ap}{2} \qquad y = \frac{0 + 4ap^2}{2}$		
$x = 2ap \qquad y = 2ap^2$		
	½	
$\therefore D(2ap, 2ap^2)$		
(ii) $x^2 = 4ay$ $\quad$ $\boxed{\text{OR}}$ $\quad x = 2ap \quad y = ap^2$		
$y = \frac{1}{4a}x^2 \qquad \frac{dx}{dp} = 2a \quad \frac{dy}{dp} = 2ap$		
$\frac{dy}{dx} = \frac{2}{4a}x \qquad \frac{dy}{dx} = \frac{dy}{dp} \times \frac{dp}{dx}$	½	
$= \frac{x}{2a}$ at $x = 2ap \qquad = 4ap \times \frac{1}{2a}$		
$m_T = \frac{2ap}{2a} \qquad =$		
$= p$		
$y - ap^2 = p(x - 2ap)$	½	
$y = px - ap^2$		
when $x = 0 \qquad y = p(0) - ap^2$	½	
$y = -ap^2$		
$B(0, -ap^2)$		

MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(iii) $m_{OB} = m_{DA}$ (O and B share the same x-ordinate ∴ vertical line) (D and A share x-ordinate) ∴ vertical	1	One pair sides = and ∥
$d_{OB} = \|-ap^2\| = ap^2$ $d_{DA} = 2ap^2 - ap^2 = ap^2$ ∴ $d_{OB} = d_{DA}$	1	
∴ ODAB ∥gm as one pair of sides is both = and ∥		
OR $m_{OD} = \frac{2ap^2}{2ap} = p = m_{AB}$ ODAB ∥gm as both pairs opp sides ∥	2	Both pairs sides ∥ all sides =
$d_{DA} = \sqrt{(2ap-2ap)^2 + (ap^2-2ap^2)^2} = \sqrt{-ap^2} = \sqrt{a^2p^4} = ap^2$	1	
$d_{OB} = \sqrt{(0-0)^2 + (0+ap^2)^2} = \sqrt{a^2p^4} = ap^2$		
$d_{OD} = \sqrt{(2ap-0)^2 + (2ap^2-0)^2} = \sqrt{4a^2p^2 + 4a^2p^4} = 2ap\sqrt{1+p^2}$ $d_{AB} = 2ap\sqrt{1+p^2}$	1	
ODAB ∥gm as opp sides =		

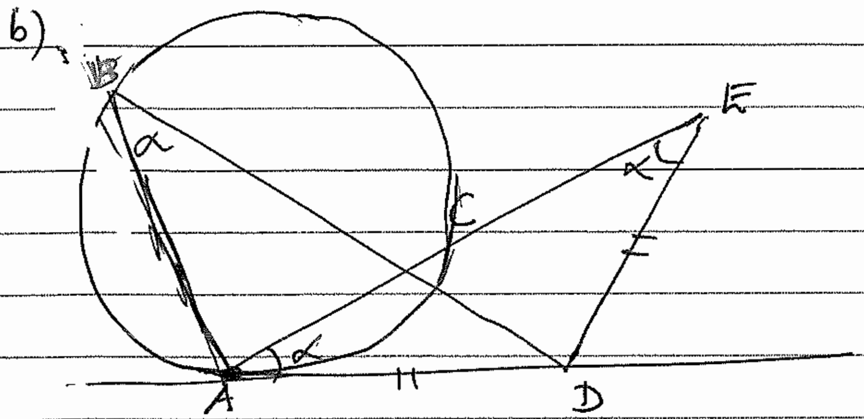
MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
midpt OA $x = ap$ $y = \frac{ap^2}{2}$	1	diagonals bisect eachother
midpt DB $x = ap$ $y = \frac{ap^2}{2}$	1	
ODAB is a //gm as diagonals bisect each other		

MATHEMATICS EXTENSION I – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $f(1.33) = 3\sin(2\times 1.33) - 1.33$	$\frac{1}{2}$	Quite well
$= 0.0596$ (4 dp)		attempted.
$f(1.34) = 3\sin(2\times 1.34) - 1.34$		However, some
$= -0.0039$ (4dp)	$\frac{1}{2}$	students
		failed to
Since $f(x)$ is a continuous function	$\frac{1}{2}$	leave their
and $f(1.33) > 0$ and $f(1.34) < 0$,	$\frac{1}{2}$	calculator in
then a root, α, exists such that		radian mode.
$1.33 < \alpha < 1.34$		The majority
		of students
		lost $\frac{1}{2}$ mark
		for not writing
		down that the
		function is
		continuous
ii) $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ $f(x) = 3\sin 2x - x$, $f'(x) = 6\cos 2x - 1$	$\frac{1}{2}$	A few
		students
$= 1.33 - \frac{3\sin(2\times 1.33) - 1.33}{6\cos(2\times 1.33) - 1}$	1	need to
		re-visit
$= 1.3394$ (4dp)	$\frac{1}{2}$	differentiation
		of
Students need to realise that when		trigonometric
solving problems involving the Bisection		functions
Method and Newton's Method with		
respect to trigonometric functions,		
the calculators should be placed in		
radian mode.		
(Degree mode – answer would		
have been 1.568462878)		
which is not correct.		

MATHEMATICS EXTENSION I – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) 		
AD = ED ∴ ∠DEA = ∠DAE (equal angles opposite equal sides of an isosceles triangle (△ADE))	1	Angles had to be identified and labelled correctly together with the reasoning otherwise
∠DAC = ∠CBA (angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment)	1	
∴ ∠ABD = ∠AED ∴ ABED is a cyclic quadrilateral (angles in the same segment are equal) or angles at the circumference standing on the same arc are equal).	1	lack of understanding of the Circle Geometry Theorems is displayed.
Some students wrote down reasons such as " ∠CAD = ∠DBE = α (angles subtended by the same arc ED) This reasoning is based on the assumption that ABED is a cyclic quadrilateral. Students were required to prove that ABED is a cyclic quadrilateral.		
Note: Spelling of isosceles triangle'		

MATHEMATICS EXTENSION I – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) $4^n + 6n - 1$ is divisible by 9		
That is, $4^n + 6n - 1 = 9M$		
Step 1 Prove true for $n = 1$		
$LHS = 4^n + 6n - 1$		
$= 4^1 + 6(1) - 1$		
$= 9$ which is divisible by 9.		
$\therefore$ true for $n = 1$		
	1mk	1mk for Step 1, 2 and the conclusion.
Step 2 Assume true for $n = k$		
That is, $4^k + 6k - 1 = 9Q$, where Q is any integer		students should be encouraged to write where Q is any integer (not restricted to positive)
$4^k = 9Q - 6k + 1$		
Step 3 Prove true for $n = k+1$		
Prove $4^{k+1} + 6(k+1) - 1$		
$= 4^k(4) + 6k + 6 - 1$		
$= 4(9Q - 6k + 1) + 6k + 6 - 1$	1	Substituting the assumption appropriately into step 3
$= 36Q - 24k + 4 + 6k + 6 - 1$		
$= 36Q - 18k + 9$		
$= 9(4Q - 2k + 1)$	1	Appropriate simplification
$= 9P$, where $P = 4Q - 2k + 1$		
$\therefore$ divisible by 9.		
Conclusion		
It is true for $n = k+1$ if it is true for $n = k$. Since it is true for $n = 1$, then it is true for $n = 2$, and hence it is true for $n = 3$, and so on. Hence, by the Principle of Mathematical Induction it holds true for all $n \geq 1$		The conclusion was written down very poorly.

MATHEMATICS EXTENSION 1 TRIAL HSC 2019 – QUESTION 15 (10 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

General comments

The instructions indicate that relevant mathematical reasoning and/or calculations should be included in the responses for Questions 11–14. Candidates are reminded that where a question is worth several marks, full marks may not be awarded for an answer, even if the answer given is correct, if no working is shown.

This is because mathematical communication and reasoning are included in the objectives and outcomes assessed by the examination.

Candidates are advised to show all their working so that marks can be awarded for some correct steps towards their answer. A simple example is when candidates have to round their answer to a certain degree of accuracy. Candidates should always write their calculator display before rounding their answer. They should only round their answer in the last step of working, not in an earlier step. Markers can then see that candidates have rounded correctly, even if the answer is not correct.

Areas for students to improve include:

- **recognising that the solution is a length and needs to be positive.**
- **paying attention to the mark value of the question and using it as a guide to the complexity of solution required.**
- **"Show that" or " Prove" questions: avoiding the omission of too many steps of the proof and communicating clearly about how they went from one step to the next.**
- **avoiding the omission of too many lines in the algebraic manipulation in an attempt to show the given result. In a 'show' question it must be clear how one line is obtained from another.**
- **showing appropriate working and not give an unsupported answer.**

(a)(i)

A particle is said to move with simple harmonic motion when the acceleration of the particle about a fixed-point is proportional to its displacement but opposite in direction.

Hence, when the displacement is positive the acceleration is negative and vice versa.

When asked to prove that, or show that a particle moves with simple harmonic motion, you simply show that the particle satisfies the equation

$$a = \ddot{x} = -n^2 x$$

where x is the displacement of the particle about a fixed-point O at time t and n is a positive constant ($n > 0$).

MATHEMATICS EXTENSION 1 TRIAL HSC 2019 – QUESTION 15 (10 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(a)(i) In this question, students needed to show that $x = 6sin2t$ is a solution to the differential equation $\ddot{x} = -n^2x$.		
$x = 6\sin 2t$		
$\dot{x} = \frac{dx}{dt} = 6\cos 2t \times 2$		
$= 12\cos 2t$		
$\ddot{x} = \frac{d^2x}{dt^2} = -12\sin 2t \times 2$		
$= -24\sin 2t$ —	1	for finding acceleration
$= -4 \times 6\sin 2t$		
$= -4x$ —	1	for correct form
$= -(2)^2 x$		
Since $\ddot{x} = -4x$ is in the form $\ddot{x} = -n^2x$ where $n = 2$, the particle is moving in simple harmonic motion.		
(ii) Period $= \frac{2\pi}{n}$		
$= \frac{2\pi}{2}$		
$= \pi$ — 1 provides correct solution		

MATHEMATICS EXTENSION 1 TRIAL HSC 2019 – QUESTION 15 (10 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(iii) when $x=3$:		
$6\sin 2t = 3$		
$\sin 2t = \frac{1}{2}$	① for correct value of t.	
$2t = \frac{\pi}{6}, \frac{5\pi}{6}, \ldots$		
$t = \frac{\pi}{12}$ when it <u>first</u> reaches $x=3$.		
when $t=\frac{\pi}{12}$: $\dot{x} = 12\cos\left(2\times\frac{\pi}{12}\right)$		
$= 12\cos\frac{\pi}{6}$		
$= 12\times\frac{\sqrt{3}}{2}$		
$= 6\sqrt{3}$ cm/s		① for correct velocity.
$\doteqdot 10.39230485$ cm/s		

• Students who used an incorrect answer for t and who demonstrated the relevant skills were not further penalised if they arrived at a correct solution for $\dot{x}$ for their incorrect t-value.
① mark – CFPA (it pays to show working and substitutions).

OR Students who found v^2 from integration where awarded full marks as long as their solution was correct.

MATHEMATICS EXTENSION 1 TRIAL HSC 2019 – QUESTION 15 (10 marks)

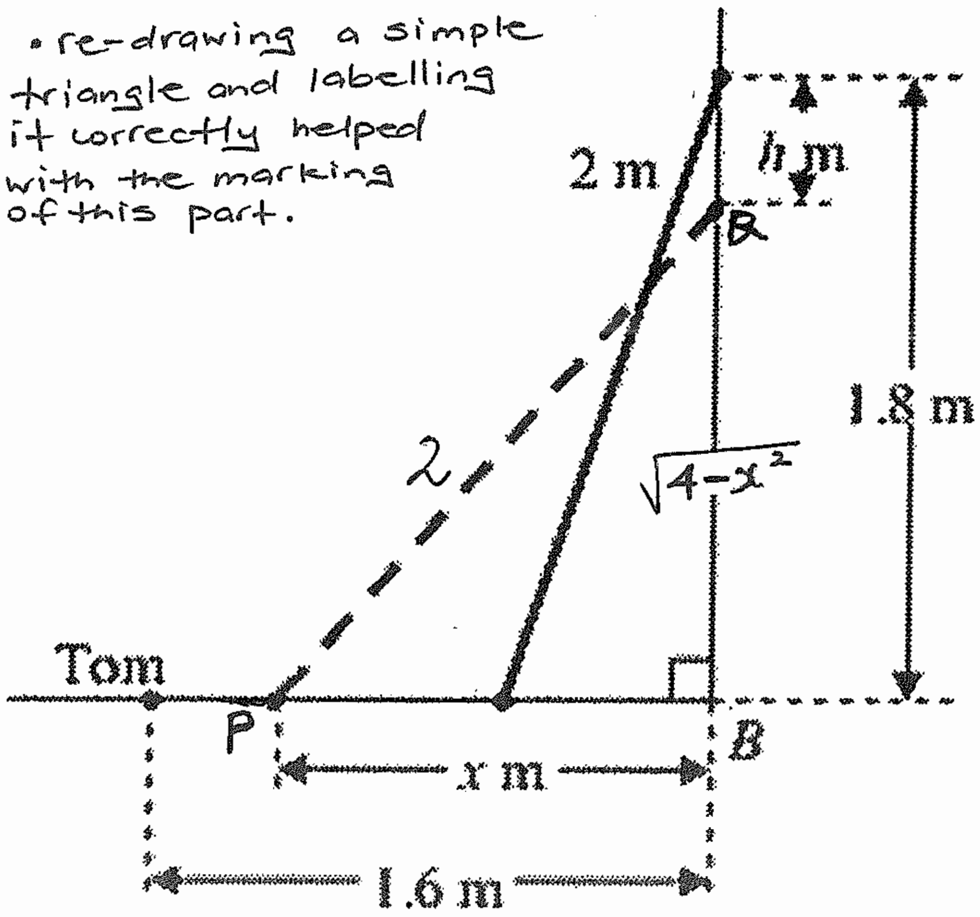
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$		
$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4x$		
$\frac{1}{2}v^2 = -\frac{4x^2}{2} + C$		
when $t=0$, $x=0$, $a=0$, $v = \dot{x} = 12\cos 2(0) = 12$		
$\frac{1}{2}(12)^2 = 0 + C$		
$\therefore C = 72$		
$\frac{1}{2}v^2 = -2x^2 + 72$		
$v^2 = -4x^2 + 144$		
when $x=3$: $v^2 = -4(3)^2 + 144 = 108$		
$v = \pm\sqrt{108} = \pm 6\sqrt{3}$		
$\therefore v = +6\sqrt{3}$ cm/s with no reference to why they chose the positive value over the negative value were awarded $1\frac{1}{2}$ marks.		
• some students backed up their answer for $v = +6\sqrt{3}$ by sketching x & v and showing that at $x=3$, $t = \frac{\pi}{12}$ and hence v is positive at this time were awarded 2 marks.		

MATHEMATICS EXTENSION 1 TRIAL HSC 2019 – QUESTION 15 (10 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

(b) (i)

• re-drawing a simple triangle and labelling it correctly helped with the marking of this part.



From Δ PQB : Using Pythagoras' Theorem :

$BQ = \sqrt{PQ^2 - PB^2}$ $\qquad BQ = \sqrt{2^2 - x^2}$

$= \sqrt{4-x^2}$ (1)

• students needed to show their use of pythagoras theorem for BQ.

$BQ + h = 1.8$

$h = 1.8 - BQ$

$\therefore h = 1.8 - \sqrt{4-x^2}$ (1)

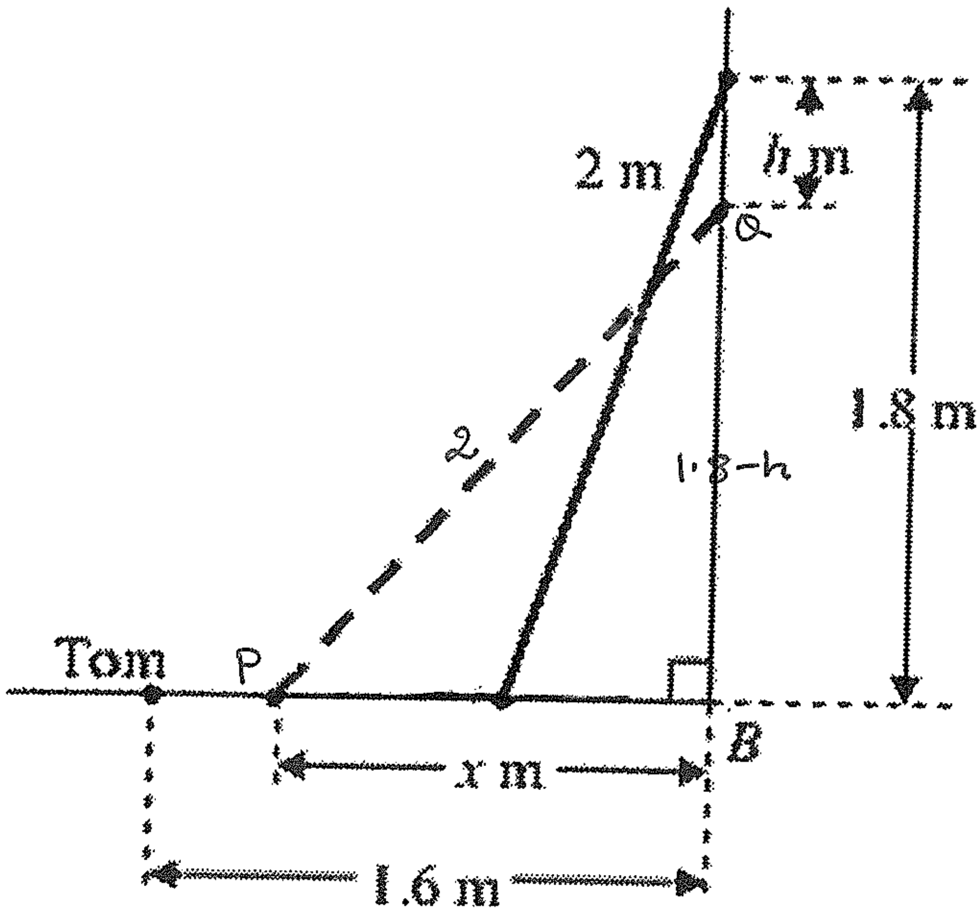
• needed to show statement that $BQ + h = 1.8$ OR $h = 1.8 - BQ$ and show their substitution for BQ.

• students who used different variables and backed this up with a clear diagram were still awarded 2 marks as long as all steps were shown.

MATHEMATICS EXTENSION 1 TRIAL HSC 2019 – QUESTION 15 (10 marks)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

OR



BQ
$= 1\cdot8 - h$

Using Pythagoras' Theorem for ΔPQB:

$$PQ^2 = BQ^2 + PB^2$$

$$2^2 = (1\cdot8-h)^2 + x^2$$

$$4 - x^2 = (1\cdot8-h)^2$$

$$\sqrt{4-x^2} = 1\cdot8 - h \quad \text{(since } x > 0\text{)}$$

$$\sqrt{4-x^2} + h = 1\cdot8$$

$$\therefore h = 1\cdot8 - \sqrt{4-x^2}$$

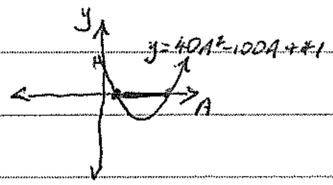
2 provides correct solution

- majority of the students used this method and were more successful with their setting out.

MATHEMATICS EXTENSION 1 TRIAL HSC 2019 – QUESTION 15 (10 marks)		
SUGGESTED SOLUTIONS	**MARKS**	**MARKER'S COMMENTS**
(b) (ii) using (i) $h = 1.8 - \sqrt{4-x^2}$; $h = 1.8 - (4-x^2)^{\frac{1}{2}}$; $\frac{dh}{dx} = -\frac{1}{2}(4-x^2)^{-\frac{1}{2}} \times -2x$; $= \frac{x}{\sqrt{4-x^2}}$ — ① for correct $\frac{dh}{dx}$		• many errors made when differentiating h w.r.t. x. • (some integrated and still wrote their answer as $\frac{dh}{dx}$) (−1) for this.
$\therefore \frac{dx}{dh} = \frac{\sqrt{4-x^2}}{x}$; $\frac{dh}{dt} = 0.5$ m/min	NOTE:	$\frac{dh}{dt} = 0.5$ m/min not 1.6.
Using chain rule: $\frac{dx}{dt} = \frac{dx}{dh} \times \frac{dh}{dt}$; $= \frac{\sqrt{4-x^2}}{x} \times 0.5$		• showing all working with substitutions made marking this question easier as I could check answers using my calculator CFPA.
When $x = 1.6$: $\frac{dx}{dt} = \frac{\sqrt{4-1.6^2}}{1.6} \times 0.5$ — ①	or equivalent merit	• Bald answers were not awarded any marks!
$= \frac{\sqrt{1.44}}{1.6} \times 0.5$; $= \frac{1.2}{1.6} \times 0.5$; $= 0.75 \times 0.5$; • all these lines were not necessary, I have put them in as my way of checking steps & calculations.		• many students forgot to reciprocate their $\frac{dh}{dx}$ (−1) leading to an answer of $\frac{2}{3}$ m/min (2 marks awarded).
$= 0.375$ m/min ; $= \frac{3}{8}$ m/min	① for correct answer	

• students who used $\frac{dh}{dt} = -0.5$ and showed all working, arrived at -0.375 m/min were awarded $(2\frac{1}{2})$ marks.

MATHEMATICS EXTENSION 1 – QUESTION 16

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) (i) $50t = x\sec\theta$		
$t = \frac{x\sec\theta}{50}$	1	1 mark for making
$y = \frac{50x\sec\theta}{50}\sin\theta - 5\left(\frac{x\sec\theta}{50}\right)^2$	1	t the subject and
		1 mark for correct
$= \frac{x\sin\theta}{\cos\theta} - \frac{5x^2\sec^2\theta}{2500}$		substitution into y
$= x\tan\theta - \frac{x^2}{500}\sec^2\theta$		
(ii) When $x = 200$ $y = 2$		
$2 = 200\tan\theta - \frac{200^2}{500}\sec^2\theta$	1	1 mark for substitution
$= 200\tan\theta - 80\sec^2\theta$		
$= 200\tan\theta - 80(\tan^2\theta + 1)$	1	1 mark for using
$= 200\tan\theta - 80\tan^2\theta - 80$		$\sec^2\theta = \tan^2\theta + 1$
$82 = 200\tan\theta - 80\tan^2\theta$		
$41 = 100\tan\theta - 40\tan^2\theta$		
$\Rightarrow 40\tan^2\theta - 100\tan\theta + 41 = 0$		
(iii) Let $A = \tan\theta$		
$40A^2 - 100A + 41 = 0$		
 $A = \frac{100 \pm \sqrt{100^2 - 4\times 40\times 41}}{80}$		
$= \frac{100 \pm \sqrt{3440}}{80}$	1	
$= \frac{25 \pm \sqrt{215}}{20}$		
$\frac{25-\sqrt{215}}{20} \leq A \leq \frac{25+\sqrt{215}}{20}$		
$\frac{25-\sqrt{215}}{20} \leq \tan\theta \leq \frac{25+\sqrt{215}}{20}$		
$27° \leq \theta \leq 63°$ to the nearest degree	1	

MATHEMATICS EXTENSION 1 – QUESTION 16

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) (i) Area of rectangle $1 \times k = k$		
Area enclosed $= k - \int_0^k \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$	1	1 mark for setting up the integral
If $f(x) = e^x + e^{-x}$		
$f'(x) = e^x - e^{-x}$		
using $\int_0^k \frac{f'(x)}{f(x)}\,dx = \left[\ln\lvert f(x)\rvert\right]_0^k$		
Area enclosed $= k - \left[\ln\lvert e^x + e^{-x}\rvert\right]_0^k$		
$e^k + e^{-k} > 0$		
$= k - \left(\ln(e^k + e^{-k}) - \ln(e^0 + e^{-0})\right)$	1	1 mark for finding the integral
$= k - \left(\ln(e^k + e^{-k}) - \ln 2\right)$		
$= k - \ln(e^k + e^{-k}) + \ln 2$		
(ii) Since $(e^k + e^{-k}) > e^k \quad (e^{-k} > 0)$		
$\ln(e^k + e^{-k}) > \ln e^k$	1	1 mark for setting this up
$-\ln(e^k + e^{-k}) < -\ln e^k$		
$k - \ln(e^k + e^{-k}) < k - \ln e^k$		
$k - \ln(e^k + e^{-k}) < k - k$	1	1 mark for completing the proof
$k - \ln(e^k + e^{-k}) < 0$		
$k - \ln(e^k + e^{-k}) + \ln 2 < \ln 2$		
$\therefore A < \ln 2$ for all $k > 0$		
All sections of Q16 were done well except for b)(ii) which was poorly done		